Cascade control

Advanced Modeling and Control

Cascade control

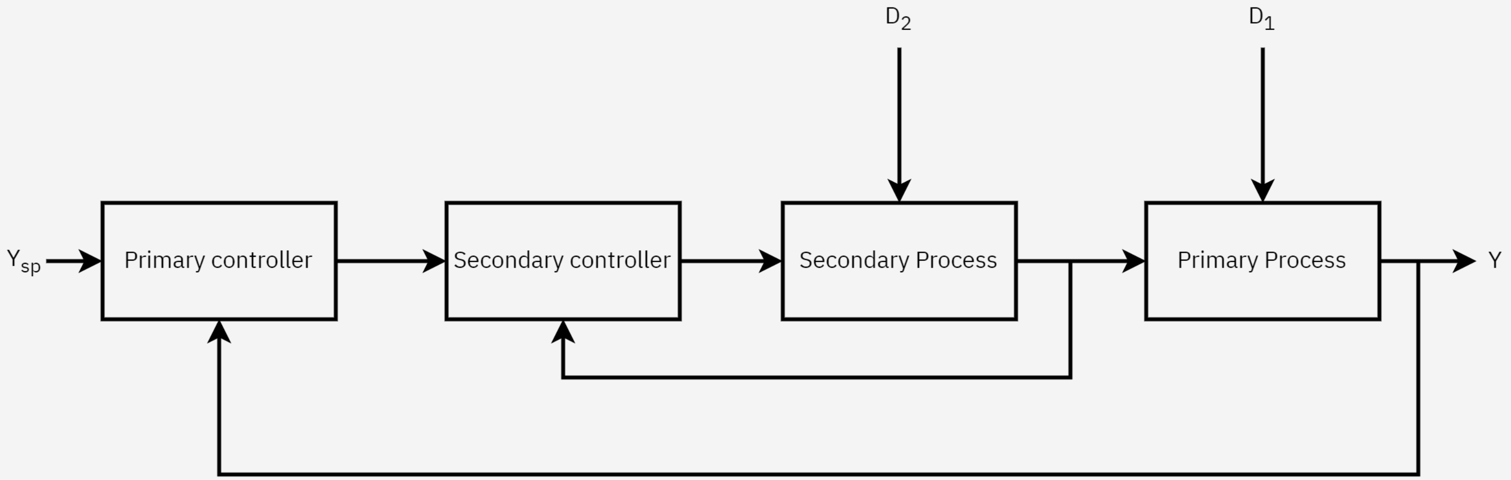
- The feedback control configuration involves one measurement (output) and one manipulated variable in a single loop.
- A disadvantage of conventional feedback control is that corrective action for disturbances does not begin until after the controlled variable deviates from the set point.
- Feedforward control offers large improvements over feedback control for processes that have large time constants or time delays.
- However, feedforward control requires that the disturbances be measured explicitly, and that a steady-state or dynamic model be available to calculate the controller output.
- An alternative approach that can significantly improve the dynamic response to disturbances employs a secondary measured variable and a secondary feedback controller.

> ⓘ In cascade control, we have **one** manipulated variable and more than one measurement.

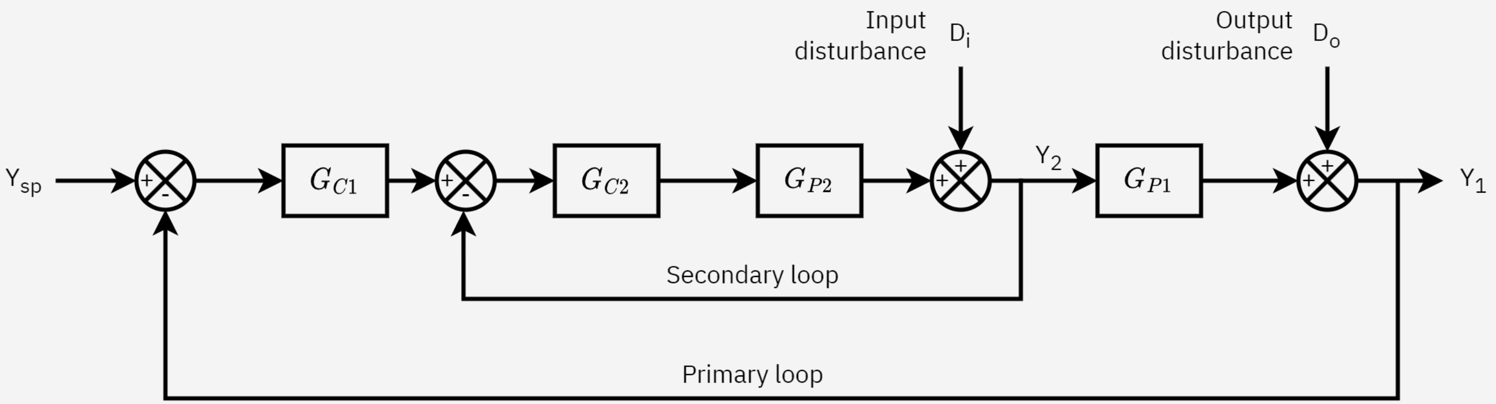
How Cascade Control Works?

- Cascade control system consists of at least 2 controllers with 1 primary loop and 1 secondary loop.
- Requires 2 measurements – 1 primary measurement and 1 secondary measurement
- Cascade control is primarily aimed to improve disturbance rejection or regulatory control performance.
- Provides early compensation of input disturbance via the secondary controller.
- Key features:
  a. The disturbance must has an effect on the secondary measurement
  b. Causal (cause-and-effect) relationship between the secondary measurement and manipulated variable
  c. Causal relationship between the manipulated variable, and between secondary and primary measurements.
  d. Secondary loop must be faster than the primary loop

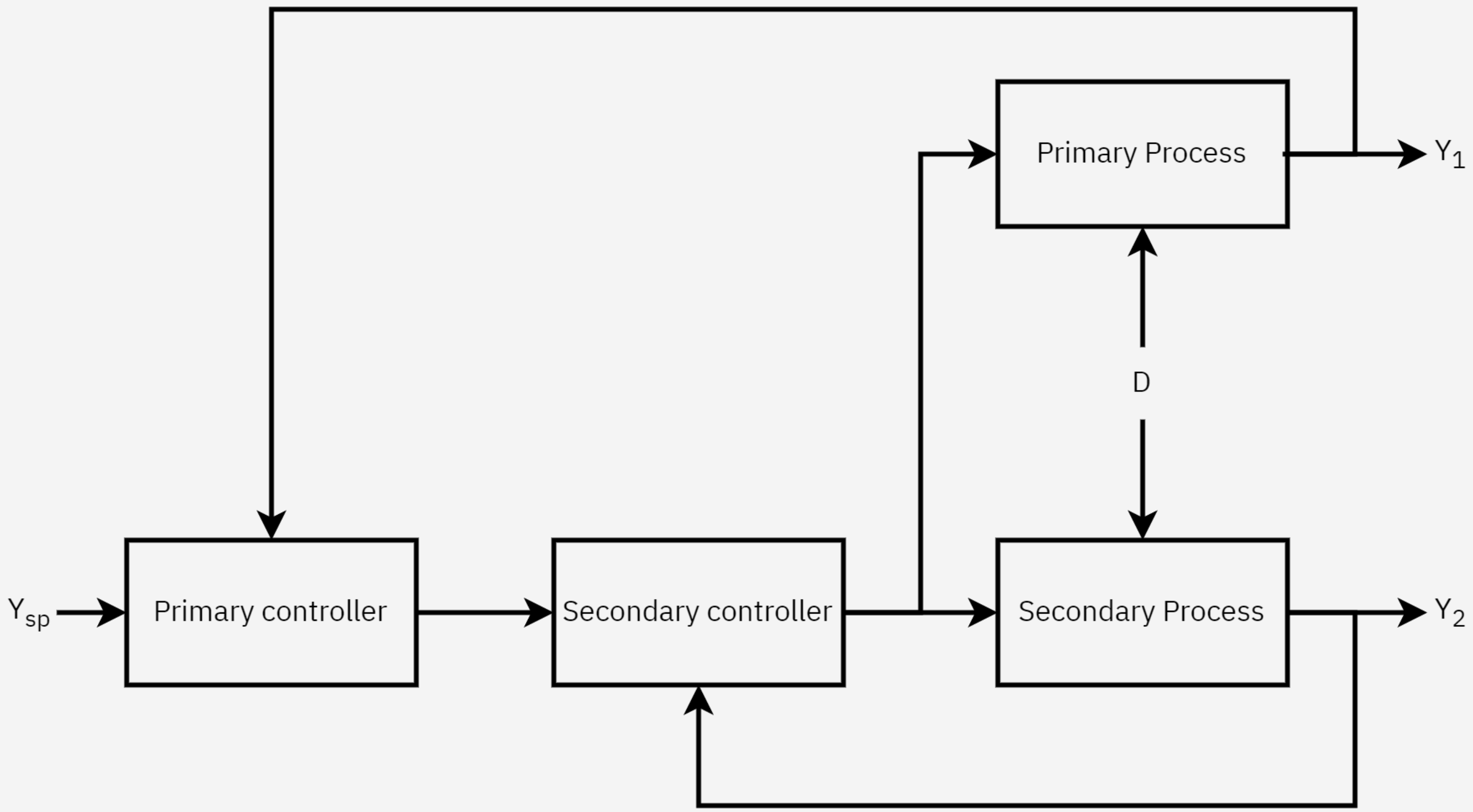
Series cascade control



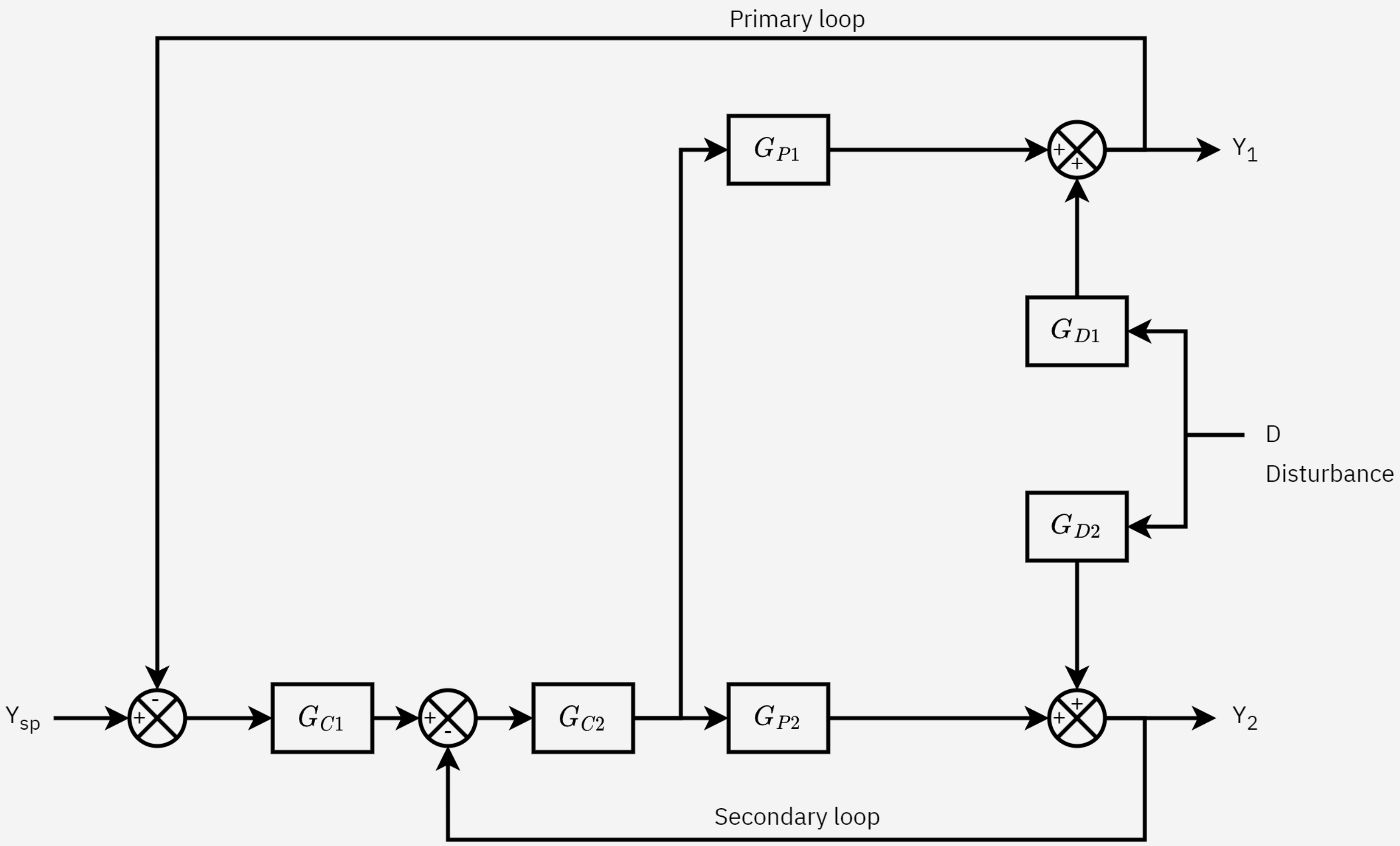
Series cascade control



Parallel cascade control



Parallel cascade control



Advantages and Disadvanteges

Advantages

- Removes effects of disturbances and improves disturbance rejection performance
- Reduces the negative effect of process nonlinearity
- Improves control performance and stability of a process with long time-delay
- Uses traditional PID-type controllers

Disadvantages

- Requires more than 1 measurements and sensors – increased cost
- More tuning parameters to handle – increased tuning task
- Potentially more wear and tear as the the inner loop is tuned aggressively

Analysis of Cascade Control System

Process models

- Open loop stable

$$G_p = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

- Integrating

$$G_p = \frac{K_p e^{-\theta s}}{s}$$

- Open loop unstable

$$G_p = \frac{K_p e^{-\theta s}}{\tau s - 1}$$

PID controllers

- PI controller

$$G_c = K_c \left(1 + \frac{1}{\tau_I s}\right)$$

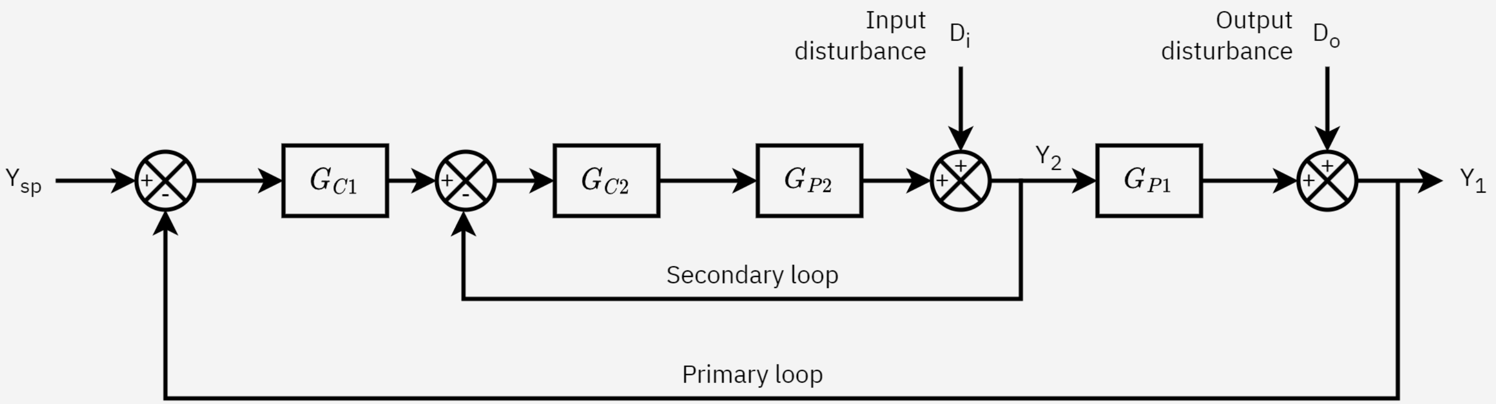
- PID controller (Ideal)

$$G_c = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

- PID controller (Parallel)

$$G_c = K_c + I \frac{1}{s} + D \frac{N}{1 + N \frac{1}{s}}$$

Stability analysis



- Primary process

$$G_{p1} = \frac{K_{p1} e^{-\theta_1 s}}{\tau_1 s + 1}$$

- Secondary process

$$G_{p2} = \frac{K_{p2} e^{-\theta_2 s}}{\tau_2 s + 1}$$

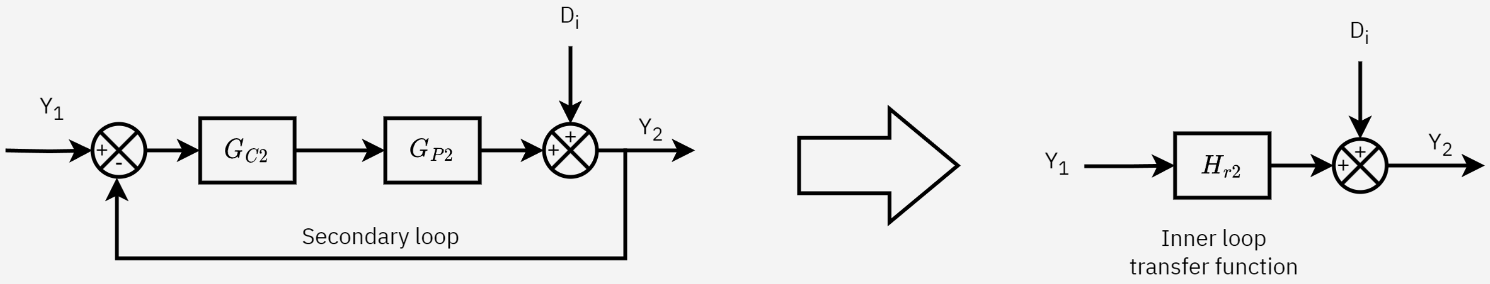
- Primary controller

$$G_c = K_c \left(1 + \frac{1}{\tau_I s}\right)$$

- Secondary controller

$$G_c = K_c$$

Inner loop analysis



- Setpoint transfer function

$$H_{r2} = \frac{G_{C2}G_{P2}}{1 + G_{C2}G_{P2}}$$

- Characteristic equation

$$1 + G_{C2}G_{P2} = 0$$

$$1 + \frac{K_{C2}K_{P2}e^{-\theta_2 s}}{\tau_2 s + 1} = 0$$

- Let Loop gain $K_{L2} = K_{C2}K_{P2}$
- Delay: $e^{-\theta_2 s} \approxeq 1 - \theta_2 s$
- Characteristic Polynomial

$$\tau_2 s + 1 + K_{L2}(1 - \theta_2 s) = 0$$

$$\underbrace{(\tau_2 - K_{L2}\theta_2)}_{a_1} s + \underbrace{(1 + K_{L2})}_{a_0} = 0$$

Inner loop analysis

- Necessary stability criterion: $a_1 > 0, a_0 > 0$
- Upper limit on the loop gain

$$a_1 = \tau_2 - K_{L2}\theta_2 > 0; \therefore K_{L2} = \frac{\tau_2}{\theta_2}$$

- lower limit on the loop gain

$$a_0 = 1 + K_{L2} > 0; \therefore K_{L2} > -1$$

- Since the lower limit is negative, due to practical reason the minimum value of loop gain should be above 0 but lower than its upper limit. Thus, for stability the loop gain is given as

$$K_{L2} = R_{p2}\left(\frac{\tau_2}{\theta_2}\right); 0 < R_{p2} < 1$$

- The parameter R_{P2} can be used to tune the controller gain as: $K_{C2} = \frac{R_{p2}}{K_{p2}}\frac{\tau_2}{\theta_2}$

Inner loop analysis

- Simplify the setpoint transfer function H_{r2} as

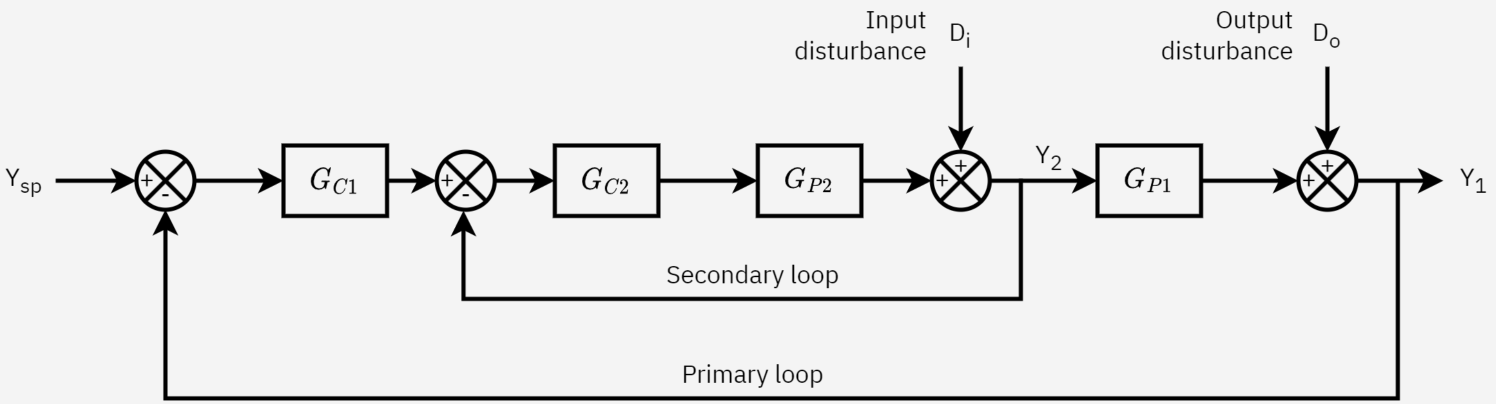
$$H_{r2} = \frac{K_O \exp\left(-\theta_2 s\right)}{\tau_{c2} s + 1}; \text{where, } K_o = \frac{K_{L2}}{1 + K_{L2}}, \quad \tau_{c2} = \frac{\tau_2}{1 + K_{L2}}$$

- Notice that $K_{L2} = R_{p2} \frac{\tau_2}{\theta_2}$
- Therefore, overall gain and closed-loop time constant can be written as

$$K_o = \frac{R_{p2} \tau_2}{\theta_2 + R_{p2} \tau_2}; \tau_{c2} = \frac{\theta_2 \tau_2}{\theta_2 + R_{p2} \tau_2}$$

> ⓘ To increase the speed of response of secondary controller, increase the value of R_{p2} but keep the value below 1 to ensure stability.

Primary loop analysis



- Augmented primary process

$$G_{pa} = H_{r2} G_{p1} \cong \frac{K_o K_{p1} e^{-(\theta_1 + \theta_2 + \tau_{c2})s}}{\tau_1 s + 1}$$

- G_{pa} is used to design or tune the primary controller. This means that the secondary controller should be designed first, as the primary design depends on the H_{r2}.

Primary loop analysis

- The effect of input disturbance is given by H_{d2}

$$H_{d2} = \frac{K_{D0}}{\tau_{c2}s+1}; K_{D0} = \frac{1}{1+K_{L2}} = \frac{\theta_2}{\theta_2 + R_{p2}\tau_2}$$

- Primary setpoint transfer function H_{r1}:

$$H_{r1} = \frac{G_{c1}H_{r2}G_{p1}}{1+G_{c1}H_{r2}G_{p1}}$$

- Characteristic equation (CE):

$$1 + G_{c1}H_{r2}G_{p1} = 0; 1 + \frac{K_{c1}K_0K_{p1}(\tau_I s+1)e^{-\theta_t s}}{\tau_I s(\tau_1 s+1)} = 0$$

where, $\theta_t = \theta_1 + \theta_2 + \tau_{c2}$

Primary loop analysis

- Let loop gain $K_{L1} = K_{C1}K_0K_{p1}$ and $e^{-\theta_t s} \approxeq 1 - \theta_t s$
- Simplifying CE to polynomial

$$\tau_I s(\tau_1 s + 1) + K_{L1}(\tau_I s + 1)(1 - \theta_t s) = 0$$

$$\underbrace{\tau_I\left(\tau_1 - K_{L1}\theta_t\right)}_{a_2}s^2 + \underbrace{\tau_I + K_{L1}\left(\tau_I - \theta_t\right)}_{a_1}s + \underbrace{K_{L1}}_{a_0} = 0$$

- Necessary stability criterion requires $a_2 > 0$, $a_1 > 0$, and $a_0 > 0$
- These provide the limits for loop gain K_{L1}
- To ensure stability, the loop gain must be bounded between its minimum upper limit and maximum lower limit
- Provides tuning parameters for the controller

Choice of Secondary Measured Variables

- There should be a well-defined relation between the primary and secondary measured variables.
- Essential disturbances should act in the inner loop.
- The inner loop should be faster than the outer loop. The typical rule of thumb is that the average residence times should have a ratio of at least five.
- It should be possible to have a high gain in the inner loop.

Summary

- Cascade control can be used when there are several measurement signals and one control variable.
- It is particularly useful when there are significant dynamics, e.g., long dead times or long time constants, between the control variable and the process variable.
- Tighter control can then be achieved by using an intermediate measured signal that responds faster to the control signal.