Introduction to digital control

In class activities

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Activities

1. Given Transform :

$$F(s) = \frac{1}{s(s+a)}, \quad a > 0$$

   find $\lim_{n\to\infty} F(nT)$.

Solution

First find the time domain function $f(t)$.

1. Inverse Laplace Transform :

   The inverse Laplace transform of $F(s)$ will give the time-domain function $f(t)$. We can use partial fraction decomposition to find it:

$$F(s) = \frac{1}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a}$$

   Solving for A and B:

$$\frac{1}{s(s+a)} = \frac{A(s+a)+Bs}{s(s+a)}$$

$$1 = A(s+a) + Bs$$

   Setting $s = 0$:

$$1 = Aa \implies A = \frac{1}{a}$$

   Setting $s = -a$:

$$1 = B(-a) \implies B = -\frac{1}{a}$$

   Therefore,

$$F(s) = \frac{1}{a}\left(\frac{1}{s} - \frac{1}{s+a}\right)$$

2. Inverse Laplace Transform in Time-Domain :

Using standard Laplace transform pairs:

$$f(t) = \frac{1}{a}\left(1 - e^{-at}\right)$$

4. Sampling the Function :

We are interested in finding $\lim_{n\to\infty} F(nT)$, which implies evaluating the limit of $f(t)$ at discrete points $t = nT$:

$$F(nT) = \frac{1}{a}\left(1 - e^{-anT}\right)$$

5. Taking the Limit :

$$\lim_{n\to\infty} F(nT) = \frac{1}{a}\left(1 - \lim_{n\to\infty} e^{-anT}\right)$$

Since $a > 0$, as $n \to \infty$, $e^{-anT} \to 0$.
Therefore, the limit becomes:

$$\lim_{n\to\infty} F(nT) = \frac{1}{a}(1-0) = \frac{1}{a}$$

6. z-transform:

   Sample $f(t)$ at intervals of T.

$$f(nT) = \frac{1}{a}\left(1 - e^{-anT}\right)$$

The z-transform of a discrete-time sequence $f(nT)$ is given by:

$$F(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}$$

Substitute the expression for $f(nT)$:

$$F(z) = \sum_{n=0}^{\infty} \frac{1}{a}\left(1 - e^{-anT}\right) z^{-n}$$

Separate the summation:

$$F(z) = \frac{1}{a}\left(\sum_{n=0}^{\infty} z^{-n} - e^{-aT} \sum_{n=0}^{\infty} \left(e^{-aT} z^{-1}\right)^{n}\right)$$

Each summation is a geometric series:

- For the first summation:

$$\sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$$

- For the second summation:

$$\sum_{n=0}^{\infty} \left(e^{-aT} z^{-1}\right)^n = \frac{1}{1 - e^{-aT} z^{-1}}$$

$$\therefore F(z) = \frac{1}{a} \left(\frac{1}{1 - z^{-1}} - \frac{e^{-aT}}{1 - e^{-aT} z^{-1}} \right)$$

2. Consider the transfer function below

$$G(s) = \frac{18}{(s+5)(s+3)} \tag{1}$$

The zero hold transfer function is given by

$$G_{ZOH} = \frac{1 - e^{-sT_s}}{s} \tag{2}$$

determine the discrete transfer function using `c2d` command for different T_s. Plot the responses.

Solution

The code is given in mlx file.

3. Consider a continuous process

$$G(s) = \frac{2(1-s)}{(3s+1)(s+1)} \tag{3}$$

convert G(s) to discrete transfer function and plot the step response. Try different hold functions.

Solution

The code is given in mlx file.

4. For the transfer function

$$G(s) = \frac{1}{s+1} \tag{4}$$

plot the effect of sample time on response.

Solution

The code is given in mlx file.

5. Design a feedback control system around the first-order plant

$$G(s) = \frac{1}{0.2s + 1} \tag{5}$$

with the requirements that

i. the steady-state error is maximum 2% for a ramp input and
ii. the phase margin is greater than 48 degrees.

Use controller transfer function and check phase margin.

$$G(s) = \frac{500s + 50}{100s^2 + s} \tag{6}$$

Solution

To design a feedback control system for the first-order plant:

- Steady-State Error for a Ramp Input

The system's steady-state error for a ramp input is determined by the type and the system gain K_v. A unity feedback system with a proportional controller can be considered:
The type of system is determined by the number of poles at the origin (s=0). This system is Type 0 because there is no pole at the origin in G(s).
K_v, the velocity error constant is given by:

$$K_v = \lim_{s \to 0} sG_{open}(s)$$

The steady-state error for a ramp input is:

$$e_{ss} = \frac{1}{K_v}$$

To satisfy $e_{ss} < 0.02$, $K_v > 50$.
Given $G(s) = \frac{1}{0.2s+1}$, if we use a proportional controller with gain K:

$$K_v = \lim_{s \to 0} s \cdot \frac{K}{0.2s + 1} = \frac{K}{0.2}$$

To achieve $K_v > 50$:

$$\frac{K}{0.2} > 50 \implies K > 10$$

Hence, the proportional gain K must be greater than 10.

- P only controller mlx file.
- PI controller mlx file.

6. Convert the controller from Equation 6 to discrete form using different sample times. Plot the Bode plot for the continuous and discrete controller and discuss the differences.

7. Design and compare continous and digital controller for the following transfer function

$$G(s) = \frac{2exp(-s)}{4s + 1} \tag{7}$$