Multivariable Centralized Control and MPC

In class activities

Ranjeet Utikar

2023-10-02

Activities

1. Explain the advantages and limitations of Model Predictive Control compared to the conventional decentralized PID control system.
2. Consider the process given by

$$G(s) = \begin{bmatrix} \frac{2\exp(-7s)}{10s+1} & \frac{0.5\exp(-4s)}{19s+1} \\ \frac{\exp(-3s)}{20s+1} & \frac{1.5\exp(-2s)}{15s+1} \end{bmatrix} \tag{1}$$

Design the decouplers D_{12} and D_{21} and comment whether the systems are physically realizable or not.

> Solution
>
> The code for calculating decoupler transfer function is given in Matlab file/ mlx file.

3. The discrete-time step response model of a process is given in Table 1.

Table 1: Discrete-time step response model

t	i	Δu	y(t)	a_i
0	0	1	0	0
1	1	0	0.3	0.3
2	2	0	0.6	0.6
3	3	0	0.7	0.7
4	4	0	0.8	0.8
5	5	0	0.86	0.86
6	6	0	0.88	0.88
7	7	0	0.89	0.89

Suppose that the process is subjected to a consecutive step changes in the input: $\Delta u = 1$ at t=0, $\Delta u = 1$ at t=2 and $\Delta u = -1$ at t=4, determine the values of y_5 and y_9.

Solution

The code for calculating decoupler transfer function is given in Matlab file/ mlx file. The data in Table 1 can be downloaded from discrete_time_response.csv.

4. Develop a DTSRM for the following transfer function

$$G_p(s) = \frac{2e^{-2s}}{5s+1} \tag{2}$$

Solution

For the given transfer cunction, - K = 2 - τ = 5 - θ = 2 seconds

Apply a Unit Step Input

To develop the step response model, apply a unit step change in the input $u(t)$:

$$\Delta u(t) = 1, \text{ for } t \geq 0$$

Let's use T_s = 1 second.

Calculate the Step Response Coefficients a_i

The step response coefficients a_i represent the fraction of the process response that occurs in each discrete time interval. The response of the system is delayed by θ seconds, so no change is observed in the output until after θ.
The response $y(t)$ to the step input for a FOPDT system is:

$$y(t) = K\left(1 - e^{-\frac{t-\theta}{\tau}}\right), \quad t \geq \theta$$

The discrete response coefficients a_i are then calculated as:

$$a_i = y(i) - y(0)$$

first few coefficients:

i	Time (s)	$y(iT_s)$	a_i
0	0	0	0
1	1	0	0
2	2	0	0
3	3	$2\left(1 - e^{-\frac{1}{5}}\right)$	0.3625
4	4	$2\left(1 - e^{-\frac{2}{5}}\right)$	0.6594
5	5	$2\left(1 - e^{-\frac{3}{5}}\right)$	0.9024
6	6	$2\left(1 - e^{-\frac{4}{5}}\right)$	1.1013

Construct the DTSRM

The DTSRM uses the coefficients a_i to predict future outputs based on past input changes:

$$y_n = y_0 + \sum_{i=1}^{n} a_i \Delta u(t_{n-i})$$

For example,

$$y(3) = y_0 + a_3 \Delta u(0) + a_2 \Delta u(1) + a_1 \Delta u(3)$$

The code for calculating a_i is given in Matlab file/ mlx file.

5. Second-Order Plus Dead-Time (SOPDT) Model to DTSRM

For the following transfer function

$$G_p(s) = \frac{K}{\tau_1 s + 1} \cdot \frac{1}{\tau_2 s + 1} e^{-\theta s}$$

develop DTSRM.

Tip

The code for calculating a_i is given in Matlab file/ mlx file.