Multivariable Centralized Control and MPC

In class activities

Ranjeet Utikar

2023-10-02

# 1. Activities

1. Explain the advantages and limitations of Model Predictive Control compared to the conventional decentralized PID control system.
2. Consider the process given by

$$G\left(s\right)=\left[\begin{matrix}\frac{2exp\left(−7s\right)}{10s+1}&\frac{0.5exp\left(−4s\right)}{19s+1}\\\frac{exp\left(−3s\right)}{20s+1}&\frac{1.5exp\left(−2s\right)}{15s+1}\end{matrix}\right]  \left(1\right)$$

Design the decouplers $D\_{12}$ and $D\_{21}$ and comment whether the systems are physically realizable or not.

|  |
| --- |
|  Solution |
| The code for calculating decoupler transfer function is given in [Matlab file](./decoupler_for_multivariable_system.m)/ [mlx file](./decoupler_for_multivariable_system.mlx). |

1. The discrete-time step response model of a process is given in [Table 1](#tbl-1).

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: Discrete-time step response model

| t | i | $Δu$ | y(t) | ai |
| --- | --- | --- | --- | --- |
| 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0.3 | 0.3 |
| 2 | 2 | 0 | 0.6 | 0.6 |
| 3 | 3 | 0 | 0.7 | 0.7 |
| 4 | 4 | 0 | 0.8 | 0.8 |
| 5 | 5 | 0 | 0.86 | 0.86 |
| 6 | 6 | 0 | 0.88 | 0.88 |
| 7 | 7 | 0 | 0.89 | 0.89 |

 |

Suppose that the process is subjected to a consecutive step changes in the input: $Δu=1$ at t=0, $Δu=1$ at t=2 and $Δu=−1$ at t=4, determine the values of y5 and y9.

|  |
| --- |
|  Solution |
| The code for calculating decoupler transfer function is given in [Matlab file](./discrete_timestep_response_model.m)/ [mlx file](./discrete_timestep_response_model.mlx). The data in [Table 1](#tbl-1) can be downloaded from [discrete\_time\_response.csv](./discrete_time_response.csv). |

1. Develop a DTSRM for the following transfer function

$$G\_{p}\left(s\right)=\frac{2e^{−2s}}{5s+1}  \left(2\right)$$

|  |
| --- |
|  Solution |
| For the given transfer cunction, - $K$ = 2 - $τ$ = 5 - $θ$ = 2 seconds1.1 Apply a Unit Step InputTo develop the step response model, apply a unit step change in the input $u\left(t\right)$:$$Δu\left(t\right)=1, for t\geq 0$$Let’s use $T\_{s}$ = 1 second.1.2 Calculate the Step Response Coefficients $a\_{i}$The step response coefficients $a\_{i}$ represent the fraction of the process response that occurs in each discrete time interval. The response of the system is delayed by $θ$ seconds, so no change is observed in the output until after $θ$.The response $y\left(t\right)$ to the step input for a FOPDT system is:$$y\left(t\right)=K\left(1−e^{−\frac{t−θ}{τ}}\right), t\geq θ$$The discrete response coefficients $a\_{i}$ are then calculated as:$$a\_{i}=y\left(i\right)−y\left(0\right)$$first few coefficients:

| $i$ | Time (s) | $y\left(iT\_{s}\right)$ | $a\_{i}$ |
| --- | --- | --- | --- |
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 2 | 2 | 0 | 0 |
| 3 | 3 | $2\left(1−e^{−\frac{1}{5}}\right)$ | 0.3625 |
| 4 | 4 | $2\left(1−e^{−\frac{2}{5}}\right)$ | 0.6594 |
| 5 | 5 | $2\left(1−e^{−\frac{3}{5}}\right)$ | 0.9024 |
| 6 | 6 | $2\left(1−e^{−\frac{4}{5}}\right)$ | 1.1013 |

1.3 Construct the DTSRMThe DTSRM uses the coefficients $a\_{i}$ to predict future outputs based on past input changes:$$y\_{n}=y\_{0}+\sum\_{i=1}^{n}a\_{i}Δu\left(t\_{n−i}\right)$$For example,$$y\left(3\right)=y\_{0}+a\_{3}Δu\left(0\right)+a\_{2}Δu\left(1\right)+a\_{1}Δu\left(3\right)$$The code for calculating $a\_{i}$ is given in [Matlab file](./dtsrm_from_fopdt.m)/ [mlx file](./dtsrm_from_fopdt.mlx). |

1. Second-Order Plus Dead-Time (SOPDT) Model to DTSRM

For the following transfer function

$$G\_{p}\left(s\right)=\frac{K}{τ\_{1}s+1}⋅\frac{1}{τ\_{2}s+1}e^{−θs}$$

develop DTSRM.

|  |
| --- |
|  Tip |
| The code for calculating $a\_{i}$ is given in [Matlab file](./dtsrm_from_sopdt.m)/ [mlx file](./dtsrm_from_sopdt.mlx). |