

MIMO Systems: Decentralized Control

In class activities

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Activities

1. Explain what is controller pairings in a decentralized control strategy.
2. Describe the factors that affect the controller pairings.
3. Explain how decoupling control work and its advantages and limitations.
4. Consider the multivariable processes below and suggest suitable controller pairings based on steady-state RGA and dynamic RGA (DRGA) for each of the process.

i. 2x2 MIMO

$$G(s) = \begin{bmatrix} \frac{5e^{-3s}}{4s+1} & \frac{2.5e^{-5s}}{15s+1} \\ \frac{-4e^{-6s}}{20s+1} & \frac{e^{-s}}{5s+1} \end{bmatrix} \quad (1)$$

RGA is given by

$$\lambda_{11} = \frac{k_{11}}{k_{11} - \frac{k_{12}k_{21}}{k_{22}}} = \frac{1}{1 - \frac{k_{12}k_{21}}{k_{11}k_{22}}} \quad (2)$$


Solution

The code for calculating RGA is given in [Matlab file/ mlx file](#).

A simulink implementation using lti system block is in [relative_gain_array_example_lti_system.slx](#).

ii. 2x2 MIMO


$$G = \begin{bmatrix} \frac{1.7e^{-2s}}{10s+1} & \frac{1.1e^{-3s}}{15s+1} \\ \frac{-1.2e^{-2.5s}}{11s+1} & \frac{1.4e^{-s}}{8s+1} \end{bmatrix} \quad (3)$$

 Solution

The code for calculating RGA is given in [Matlab file/ mlx file](#).

iii. Steady-state gain matrix


$$K = \begin{bmatrix} 0.7 & 0.3 & -0.4 \\ 1.1 & -0.6 & -0.2 \\ 0.2 & 0.5 & -0.9 \end{bmatrix} \quad (4)$$

 Solution

The code for calculating RGA is given in [Matlab file/ mlx file](#).

iv. 3x3 MIMO


$$G(s) = \begin{bmatrix} \frac{119e^{-5s}}{21.7s+1} & \frac{153e^{-5s}}{337s+1} & \frac{-2.1e^{-5s}}{10s+1} \\ \frac{37e^{-5s}}{500s+1} & \frac{76.7e^{-5s}}{28s+1} & \frac{-5e^{-5s}}{10s+1} \\ \frac{93e^{-5s}}{500s+1} & \frac{-66.7e^{-5s}}{166s+1} & \frac{-103.3e^{-5s}}{23s+1} \end{bmatrix} \quad (5)$$

 Solution

The code for calculating RGA is given in [Matlab file/ mlx file](#).

v. 2x2 MIMO

$$\begin{aligned} \mathbf{G}_{11}(s) &= \frac{1.0}{100s+1} & \mathbf{G}_{12}(s) &= \frac{0.3}{10s+1} \\ \mathbf{G}_{21}(s) &= \frac{-0.4}{10s+1} & \mathbf{G}_{22}(s) &= \frac{2.0}{100s+1} \end{aligned}$$

 Solution

The code for calculating RGA is given in [Matlab file/ mlx file](#).

5. Consider the process given by

$$G(s) = \begin{bmatrix} \frac{2 \exp(-7s)}{10s+1} & \frac{0.5 \exp(-4s)}{19s+1} \\ \frac{\exp(-2s)}{20s+1} & \frac{1.5 \exp(-3s)}{15s+1} \end{bmatrix} \quad (6)$$

Design the decouplers D_{12} and D_{21} and comment whether the systems are physically realizable or not.

💡 Solution

Decoupler D_{12} design

$$D_{12} = -\frac{g_{12}}{g_{11}}$$

$$D_{12} = -\frac{\frac{0.5e^{-4s}}{19s+1}}{\frac{2e^{-7s}}{10s+1}}$$

$$D_{12} = -\frac{0.5e^{-4s}}{19s+1} \times \frac{10s+1}{2e^{-7s}}$$

$$D_{12} = -\frac{0.5(10s+1)}{2(19s+1)} \times e^{3s}$$

$$D_{12} = -\frac{0.25(10s+1)e^{3s}}{19s+1}$$

The presence of e^{+3s} indicates that the decoupler has a predictive capability – it can predict the future input change that affects the current output, i.e., the output change first before the input change (this can't happen in our physical world). Therefore, the decoupler is considered a non-causal system, thus cannot be realized physically.

Decoupler D_{21} design

$$D_{21} = -\frac{g_{21}}{g_{22}}$$

$$D_{21} = -\frac{\frac{e^{(-2s)}}{20s+1}}{\frac{1.5e^{(-3s)}}{15s+1}}$$

$$D_{21} = -\frac{0.67(15s+1)e^{-s}}{20s+1}$$


The decoupler is physically realizable, i.e., order of numerator is not higher than the order of denominator, and also there is no predictive term, only a delay term in the decoupler.

6. Wood Berry Column controller design

The control objective in the WB distillation column is to maintain the desired compositions in both the overhead (distillate) and bottom products by manipulating the reflux flow rate and steam flow rate. However, since both inputs affect both outputs, decoupling and interaction management are often required to ensure efficient and stable control of the system.

This model is widely used in process control to study interactions in distillation columns, and it simplifies the control of more complex chemical processes.

$$G = \begin{bmatrix} \frac{12.8 \exp(-s)}{16.7s+1} & \frac{-18.9 \exp(-3s)}{21s+1} \\ \frac{6.6 \exp(-7s)}{10.9s+1} & \frac{-19.4 \exp(-3s)}{14.4s+1} \end{bmatrix} \quad (7)$$

 Solution

The code for calculating RGA and controller pairing is given in [Matlab file/ mlx file](#).
A simulink implementation using `lti` system block is in [wood_barry.slx](#).