Mid semester class test

CHEN4011 - Advanced Modelling and Control

Name: Student ID:

Date:	2025-08-25
Time:	12:00 - 14:00
Assessment duration:	90 minutes + 10 minutes reading time
Total marks:	40

Instructions to students

- **THIS IS A OPEN BOOK INVIGILATED IN CLASS TEST.**
- ***Mobile phones*** or any other devices capable of communicating information are **prohibited** from use during examinations. **You may use your laptops**.
- You may refer to the course website, Matlab documentation, and resources provided during class/workshops. Searching the internet for answers or using AI/LLM tools is strictly prohibited.
- If required, assume necessary data and clearly state and justify your assumptions.
- You must write your answers on the printed question paper provided. **Only what is written there will be marked**.
- **Syllabus**: All topics covered up to Week 5, including feedforward and ratio control, cascade control, other advanced control techniques, and time series modeling and analysis.
- **Question types**: Theory questions, problems, and Matlab/Simulink based questions. The test is not overly reliant on Matlab/Simulink. One or two questions (out of five) will require Matlab/Simulink.
- You are required to upload the Matlab/Simulink file on to blackboard:
  - Rename the file as STUDENTID_midsem_test.(m/slx)
  - Upload it using assessment submission link on blackboard.
- Any breaches of policy will be treated as cheating and addressed under the University's academic misconduct procedures.
- **YOU ARE REQUIRED TO ANSWER ALL QUESTIONS**

Question 1 **(10 marks)**

Consider the following transfer functions for a mixing process:

$$G_P(s) = \frac{4e^{-2s}}{3s+1}, \qquad G_D(s) = \frac{-6e^{-s}}{(s+1)(2s+1)}$$

The system is controlled with a PI controller of the form

$$G_C(s) = K_c\left(1 + \frac{1}{\tau_I s}\right),$$

with parameters $K_c = 20,\ \tau_I = 4$.

Assume unity measurement. The feedforward signal is added at the plant input. For parts (c)–(e), ignore the time delay.

Tasks:

a) Derive the expression for the ideal feedforward controller $G_{FF}(s)$. (2 marks)

Answer

The ideal feedforward controller is obtained from

$$G^{\star}_{FF}(s) = -\frac{G_D(s)}{G_P(s)}.$$

$$G^{\star}_{FF}(s) = -\frac{-6e^{-s}/[(s+1)(2s+1)]}{4e^{-2s}/(3s+1)} = \frac{3}{2}\,e^{+s}\,\frac{3s+1}{(s+1)(2s+1)}.$$

b) Comment on whether it is realizable. If the ideal feedforward controller is not realizable, suggest practical strategies to approximate it. (2 marks)

Answer

- The term e^{+s} corresponds to a time advance (prediction), which is non-causal and thus not physically realizable.
- Practical approximations:
  - Delay matching: Insert a deliberate 1 s delay to cancel the advance:
$$G_{FF}(s) = \frac{3}{2}\,\frac{3s+1}{(s+1)(2s+1)}\,e^{-s}.$$
  - Static gain feedforward: Use steady-state gain ratio with a filter:
$$G_{FF}(s) \approx -1.5\,\frac{1}{\tau_f s+1}.$$
  - IMC-style filtered inversion:
$$G_{FF}(s) = -\frac{G_D(s)}{G_P(s)}\,Q_{ff}(s),$$
    with $Q_{ff}(s) = \big(1/(\tau_f s+1)\big)^n$ to enforce causality and robustness.

c) Derive the open-loop transfer function $G_{OL}(s)$. (2 marks)

Answer

Ignoring delays:

$$G_P(s) = \frac{4}{3s+1}, \quad G_C(s) = 20\left(1 + \frac{1}{4s}\right).$$

$$G_{OL}(s) = G_C(s)G_P(s) = \frac{20(4s+1)}{s(3s+1)}.$$

d) Derive the closed-loop transfer function $\frac{Y(s)}{Y_{sp}(s)}$ assuming unity feedback. (2 marks)

Answer

With unity feedback:

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)} = \frac{20(4s+1)}{s(3s+1) + 20(4s+1)}.$$

Simplified:

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{80s + 20}{3s^2 + 81s + 20}.$$

e) Using the Routh criterion, comment on the stability of the closed-loop system. (2 marks)

Answer

Characteristic polynomial:

$$3s^2 + 81s + 20.$$

- All coefficients are positive.
- For a second-order system, positivity of coefficients ensures stability.
- The discriminant $81^2 - 4 \cdot 3 \cdot 20 = 6321 > 0$, confirming real negative roots.

Conclusion: The closed-loop system is stable.

Question 2 **(10 Marks)**

A binary distillation column has a total condenser and a reflux drum. The overhead vapor is condensed to liquid and split into reflux L back to the column and distillate D to product. The top-tray temperature T_{tray} used as an inferential measure of distillate composition.

The plantwide policy is to hold a constant reflux-to-distillate ratio ($R = L/D = 1.5$) to stabilize top composition x_D during feed disturbances while the drum level is regulated. A ratio station computes the reflux-flow setpoint from the measured distillate flow.

a). For ratio control identify the following (5 marks)

Controlled variable (CV) of the ratio loop:

Answer:

$r = \dfrac{L}{D}$ held at set value $R = 1.5$

Measured variables used by the ratio station:

Answer

Distillate flow D_{meas} (optionally a trim signal from T_{tray} or x_D analyzer)

Manipulated variable (MV):

Answer

Reflux flow controller setpoint L_{sp} (reflux valve)

Ratio relationship implemented (write the exact formula the ratio station should use, including an optional trim b):

Answer

$$L_{\text{sp}} = R\,D_{\text{meas}} + b$$

Major disturbances the ratio loop helps reject (list any three):

Answer

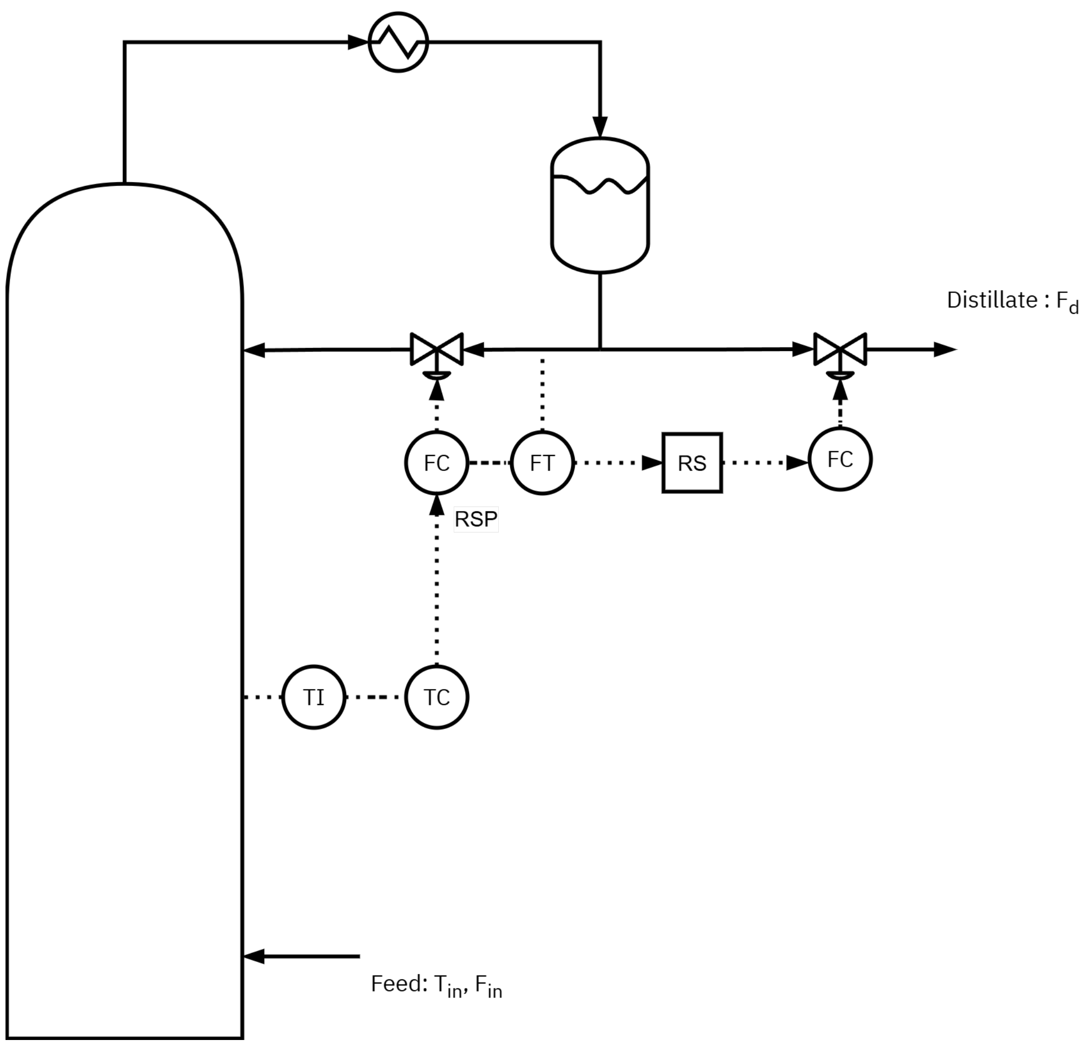
Major disturbances the ratio loop helps reject (any three):
- Feed flow-rate changes
- Feed composition changes
- Feed temperature/enthalpy changes
- Upstream pressure fluctuations
- Condenser cooling-water temperature changes

Regulatory control for safe operation (name the variables typically held and their actuators):

Answer

Regulatory control for safe operation (variables and actuators):
- Reflux drum level → distillate product valve
- Column pressure → condenser duty or vent/blanket valve
- Reboiler (sump) level → bottoms product valve
- Reboiler heat input → steam or heat-medium valve
- Feed flow → feed control valve

b). Draw a clean schematic showing a cascade control structure in which the tray-temperature controller is the primary loop and a reflux-to-distillate ratio controller is the secondary loop feeding the reflux flow controller. (5 marks)



Question 3 **(10 Marks)**

Consider the process

$$G_p(s) = \frac{5\,e^{-3s}}{(4s+1)(s+1)}.$$

A PI controller is used with parameters

$$K_c = 0.2, \qquad \tau_I = 4.$$

Develop a SIMULINK model to simulate:

1. Standard feedback control with PI only.
2. Smith Predictor based control. Use the internal model without delay,

$$G_m(s) = \frac{5}{(4s+1)(s+1)},$$

and place the actual time delay only in the forward plant path.

Use the following conditions while developing the simulation.

- Total simulation time: 100 s
- Set point: unit step at $t = 0$
- Use the PI parameters above for both cases
- Measure output $y(t)$

Upload the developed SIMULINK model to the submission link on blackboard.

Populate Table 2.

Table 2: Simulation results

Performance metric	PI only	Smith Predictor
Settling time (within 2% of nominal value)	44.6	14.7
Overshoot	44.43 %	0
Undershoot	19.16%	0
ISE	6.286	2.5

Question 4 (10 Marks)

A DC motor is driven by an input voltage and both the angular position and angular velocity are measured as outputs. The dataset `dcmotordata` is provided with MATLAB System Identification Toolbox. It contains:

- u: armature voltage (input)
- y: [angular position, angular velocity] (outputs)

Load the data in MATLAB with the commands:

```
load dcmotordata
z = iddata(y,u,0.1);
```

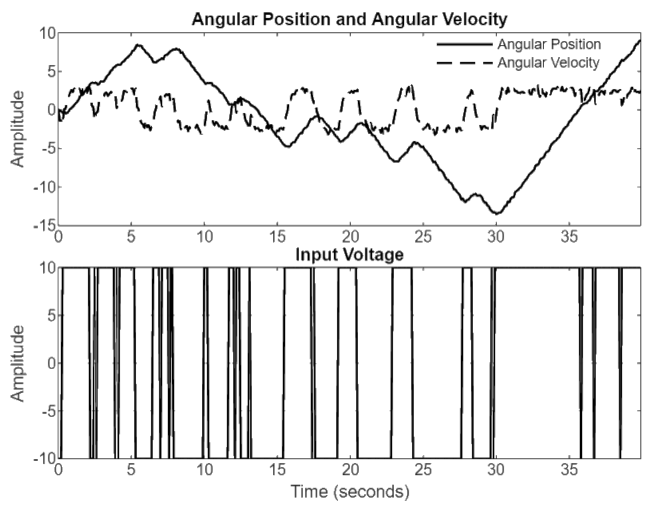


Figure 1: DC motor dataset

Here, `y` includes two outputs and `0.1` is the sampling time (s).

Use the first 200 samples of the dataset (`z(1:200)`) for model estimation and the last 200 samples (`z(201:400)`) for model prediction (testing).

Estimate the following multi-output models using MATLAB functions:

- Transfer function models with one and two poles (`tfest`)
- ARX models with orders (na=2, nb=2, nk=1) and (na=4, nb=4, nk=1) (`arx`)
- ARMAX model with orders (na=2, nb=2, nc=2, nk=1) (`armax`)

(a) Compare each model against the test data using the `compare` command. Report the fit percentage for prediction data in Table 3 for both outputs. (5 marks)

Table 3: Model fit for DC motor data

Model	Fit to angular position data	Fit to angular velocity data
Transfer function with one pole		
Transfer function with two poles		
ARX model (na=2, nb=2, nk=1)		
ARX model (na=4, nb=4, nk=1)		
ARMAX model (na=2, nb=2, nc=2, nk=1)		

Answer

Model	Fit to angular position data	Fit to angular velocity data
Transfer function with one pole	91.22	84.47
Transfer function with two poles	98.35	84.49
ARX model (na=2, nb=2, nk=1)	97.37	83.44
ARX model (na=4, nb=4, nk=1)	97.78	84.38
ARMAX model (na=2, nb=2, nc=2, nk=1)	98.26	84.82

(b) Comment on the results. (5 marks)

- Which output is harder to model?

Answer

The angular velocity output is consistently more difficult to model. Across all models, its fit percentage remains lower (≈83–85%) compared to the angular position output (≈91–98%). This suggests that the angular velocity exhibits more noise or faster dynamics that are not fully captured by the identified models.

- Does increasing model complexity always improve prediction? Explain why or why not.

No, increasing complexity does not guarantee better prediction.

For angular position, moving from a one-pole transfer function to a two-pole or higher-order ARX/ARMAX improves the fit significantly.

For angular velocity, however, increasing the order of ARX or moving to ARMAX yields only marginal improvements (less than 1%).

More complex models risk overfitting the estimation data, capturing noise rather than system dynamics, which does not translate into better performance on unseen test data.

Conclusion: A moderate-complexity model such as the ARMAX (2,2,2,1) provides a good balance—achieving excellent accuracy for angular position and the best (though still limited) accuracy for angular velocity without unnecessary over-parameterization.

Formula sheet

Ideal PI Controller Tuning

$$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$K_c = \frac{1}{K_P \tau_{ratio}}$. Typically $1 < \tau_{ratio} < 5$; $\tau_I = \tau_P$. Matlab controller parameters: $P = K_c$; $I = 1/\tau_I$.

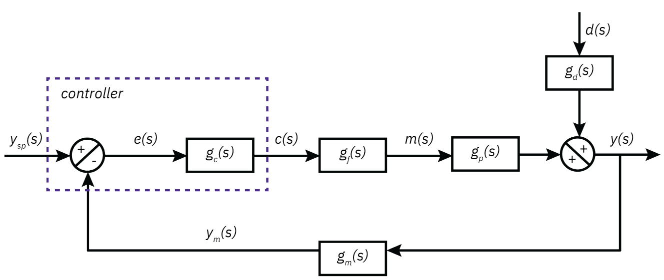
First-Order plus Deadtime (FOPDT) model

$$G(s) = \frac{K_p e^{-\theta s}}{\tau_p s + 1}$$

Idealized Feedforward Controller

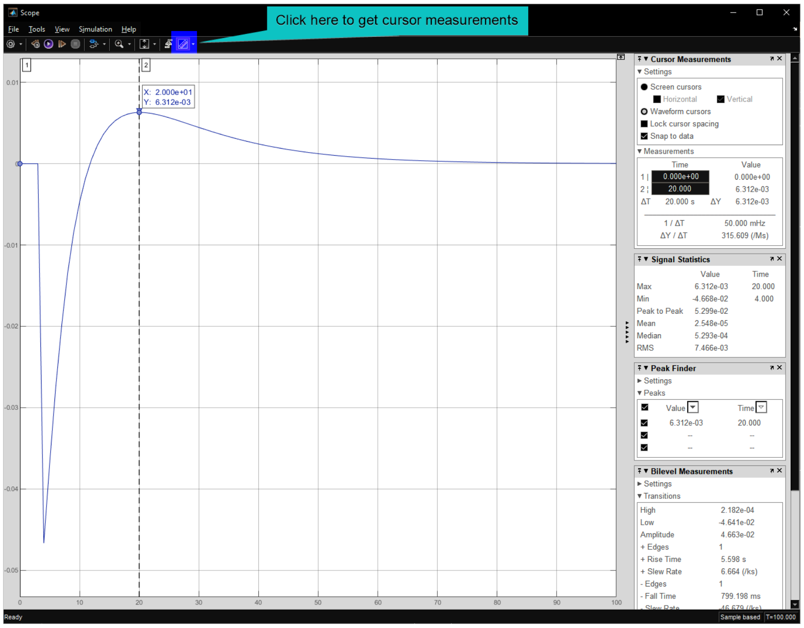
$$G_{FF} = -\frac{G_D}{G_P}$$

Simple feedback loop with output disturbance



Cursor measurements

Useful information about the time series can be obtained using cursor measurements.


Click here to get cursor measurements
X: 2.000e+01
Y: 6.312e-03
Cursor Measurements
Signal Statistics
Peak Finder
Bilevel Measurements